ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key messages

Candidates should ensure that, for a question that does not allow the use of a calculator, such as **Question 2**, sufficient detail is shown. A check should also be made to ensure that all the demands of the question have been met. Where exact answers have been asked for, answers given in decimal form will not obtain a final accuracy mark. Where answers to a specific level of accuracy have been asked for within a particular question, these should be given as a final answer, with care taken not to prematurely approximate values in the working of the solution. Where an apparently simple question has been asked, for example **Question 4(a)**, candidates should realise that this is meant to help them with a subsequent question part. The word ‘Hence’ will not always be used.

General comments

There was a wide range of marks gained over the paper. It was evident that many candidates had been well prepared for the examination by centres and had also worked hard to cover and understand the requirements of the syllabus. However, there were also some candidates who clearly had not covered the syllabus and were thus unprepared for the examination.

It should be noted that any blank pages at the end of the examination booklet may be used for solutions to questions that require extra space for some reason. It was pleasing to see some candidates make use of the blank page before using supplementary sheets.

Comments on specific questions

Question 1

Most candidates formed a quadratic equation by equating the line and curve equations. There were both algebraic and arithmetic slips in the simplification process by some candidates. Very few candidates did not make use of the discriminant of their quadratic equation and attempt to solve the resulting quadratic equation in k in order to find the critical values associated with the discriminant. Those candidates who obtained the correct critical values usually used them appropriately to obtain the correct values of k.

Question 2

Most candidates gave sufficient detail in their solution to show that a calculator had not been used. The application of the quadratic formula to the given equation was expected and the majority of candidates did this correctly, with very few sign errors seen. The discriminant was usually calculated correctly, with sufficient detail, to obtain a value of 49. Two correct solutions of $\frac{2-2\sqrt{3}}{6-10\sqrt{3}}$ and $\frac{-12-2\sqrt{3}}{6-10\sqrt{3}}$ or equivalent were common. Rationalisation was attempted by most candidates; however, some did not show sufficient detail to preclude the use of a calculator and some candidates showed no attempt at rationalisation at all, giving the two simplified solutions. Candidates who showed insufficient detail in the rationalisation process were unable to gain the final 3 marks,

Question 3

(a) It was important that candidates realised that the value of *b* had to be found first, making use of the period of the trigonometric expression. This was done successfully by many. Some candidates misinterpreted the information given, assuming that the two given points were maximum and minimum points and calculating an amplitude from this. Unfortunately, these candidates were unable to obtain more that the mark for the value of *b*. It was expected that two simultaneous equations, using the given coordinates and a value for *b*, be formed and solved to give the values of *a* and *c*.

(b) There were several different approaches that could be used to find the coordinates of the minimum point on the curve. Most candidates successfully used calculus to find at least one *x*-coordinate of a stationary point. Most candidates either used the second derivative or observation to determine the *x*-coordinate of the minimum stationary point, usually with great success. It should be noted that a candidate with incorrect values for *a* and *c*, was able to earn 3 marks in this part of the question.

Another equally successful approach involved recognising the minimum value of $\sin bx$ is –1 and working from there to obtain the *x*-coordinate of the minimum point and the corresponding *y*-coordinate. Again, a candidate with incorrect values was able to obtain 3 marks by using a correct process.

Another less common method, but equally successful, involved consideration of the sine cycle and the fact that a minimum will occur $\frac{3}{4}$ of the way through the cycle that is, when $x = 12\pi$.

Question 4

(a) A simple use of the lowest common multiple of the denominators of the two given fractions was used by most to obtain the given result. It was acceptable for a single fraction with a denominator of $(2x-1)^3$ to be formed initially, providing sufficient detail was given of the simplification process. It was intended that this part of the question help candidates re-write the integrand in **part (b)**.

(b) Unless candidates realised that they had to make use of **part (a)** and re-write the integrand as $\frac{1}{2x-1}+\frac{4}{(2x-1)^2}$, no correct progress could be made. Of those candidates who made the connection with **part (a)**, most identified that $\int \frac{1}{2x-1}\,\mathrm{d}x$ was a form of $\ln(2x-1)$. Candidates had more difficulty with $\int \frac{4}{(2x-1)^2}\,\mathrm{d}x$, with some candidates thinking that another logarithmic function was involved. For candidates whose integral was in a correct form, most were able to obtain the mark for the substitution of the limits. A specific form of answer was required and too many candidates were unable to gain a final accuracy mark for an answer of $\frac{1}{2}\ln 3$ rather than $\ln\sqrt{3}$ as required.

Question 5

(a) Most candidates were able to differentiate the given function correctly. A few candidates were unable to differentiate $\ln\left(2x^2-3\right)$ correctly, but most applied the quotient rule correctly. It was acceptable for candidates to differentiate as a product, but those that attempted this, often made errors in expressing $(3x)^{-1}$ correctly before attempting differentiation. Many candidates attempted to simplify their derivative, often making errors in this subsequent simplification. This then affected **parts (b)** and **(c)**.

(b) Most candidates realised that a substitution of $x = 2$ into their derivative was required, followed by multiplication by h, applying the small changes rule correctly. Incorrect simplification of **part (a)** meant that some candidates were unable to obtain the accuracy mark.

(c) As in **part (b)**, some candidates were able to obtain a method mark for applying the rate of change rule correctly, having simplified **part (a)** incorrectly. Some candidates were unable to obtain the accuracy mark due to premature approximation of their $\frac{dy}{dx}$ when $x = 2$.

Question 6

A completely unstructured question, candidates were expected to use their problem-solving skills and decide what steps were needed to reach the final solution. Most candidates found the value of y when $x = \frac{\pi}{12}$. Most candidates attempted to find $\frac{dy}{dx}$, but a common error was to write the derivative as $3\sec^2 x$, rather than the correct $3\sec^2 3x$. Some candidates were also unaware that $\sec^2 3x = \frac{1}{\cos^2 3x}$. In spite of these errors, most candidates attempted to find the equation of the normal and make the appropriate substitutions to obtain the coordinates of the points Q and R. Whilst many candidates were able to attempt to find the area using a correct approach, drawing a sketch of the situation would have benefitted some candidates when working out the horizontal distance between the points Q and R. Candidates who used the determinant method of finding the area were usually successful although, in some cases, a factor of $\frac{1}{2}$ was missing.

Question 7

Another completely unstructured question which required candidates to realise that they needed to integrate twice, as well as considering an arbitrary constant in each integration. Most candidates were able to obtain $\int (2-3x)^{-\frac{1}{3}}\,dx$ in a form involving $(2-3x)^{\frac{2}{3}}$, with any errors usually involving signs or multiples. In most cases, an arbitrary constant was also included in the integration although too many candidates used incorrect information to obtain the first arbitrary constant. Most candidates were able to obtain $\int (2-3x)^{\frac{2}{3}}\,dx$ in a form involving $(2-3x)^{\frac{5}{3}}$, with any errors usually again involving signs or multiples. In most cases, a second arbitrary constant was also included in the integration with correct information being used to determine its value. Many completely correct solutions were seen,

Question 8

(a) Many candidates were unaware that they needed to find the magnitude of the direction vector and compare it with the given speed of 58 in order to find the velocity vector.

(b) Most candidates were aware that they needed to make use of their velocity vector from **part (a)** in order to find the position vector of the particle A at time t.

(c) Most candidates recognised that a subtraction of the position vector found in **part (b)** with the given vector was required. Some subtractions were in the incorrect order.

(d) The application of Pythagoras' theorem to find the magnitude of the displacement vector found in **part (c)** was attempted by most candidates, making use of their displacement vectors. In some instances, the squares of the factors were subtracted rather than added.

(e) Many correct solutions were seen, although some candidates did not give their final solution to the correct level of accuracy as required. Some candidates also gave the negative solutions as well. This meant that the final accuracy mark could not be awarded. For those candidates whose quadratic equation was incorrect (usually from an error in **part (a)**), a method mark was only awarded if a correct method of solution (use of the quadratic formula) was seen.

Question 9

(a) (i) Most candidates obtained the correct value for *a*.

(ii) Few correct ranges were seen. Candidates are expected to know the properties of the graphs of logarithmic functions. The expectation is that the correct notation is also used when describing the range of a function.

(iii) The inverse function was found correctly by most candidates. Again, the expectation was that the correct notation be used in the final answer. Many candidates did not give the range of the **inverse** function and those that did often used incorrect notation or had an incorrect answer. Very few correct ranges were seen.

(iv) Responses were varied, ranging from some excellent correct sketches with all the relevant detail clearly added, to poorer sketches where it was evident that the candidates were not familiar with the shapes of the graphs of logarithmic and exponential functions. It should be noted that the exact values of the intercepts with the axes were requested. Some candidates gave rounded decimals instead.

(b) Many completely correct solutions were seen. Candidates appreciated the meaning of the notation $g^2(x)$ and hence wrote down a correct equation of $\left(x^{\frac{1}{2}}-4\right)^{\frac{1}{2}}-4=-2$. Some candidates were unable to solve this equation correctly, making algebraic errors and some candidates gave an incorrect answer of $2\sqrt{2}$ to a correct simplified equation of $x^{\frac{1}{2}}=8$.

Question 10

(a) There were many completely correct solutions to this question. Most candidates identified the common difference of the arithmetic progression correctly and went on to form an equation using the sum of the arithmetic equation which reduced to $\sin 3x=\frac{1}{2}$. Whilst some candidates only gave one correct solution to this equation, many gave the two required solutions, which were in the exact form as requested.

(b) (i) Most candidates identified the common ratio of the given geometric progression as $\frac{1}{2}\cos y$. This gained an accuracy mark. Few candidates obtained the second accuracy mark as an insufficient explanation was given as to why the progression had a sum to infinity. It was insufficient to state that $\left|\frac{1}{2}\cos y\right|<1$, or equivalent. It was necessary to state that $-\frac{1}{2}\leqslant\frac{1}{2}\cos y\leqslant\frac{1}{2}$ and comment appropriately.

(ii) Many correct responses were seen, although some candidates did not give their final answer to the required level of accuracy. Any errors were usually due to the use of an incorrect common ratio.

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

In order to succeed in this examination, candidates need to demonstrate that they are able to solve problems by applying combinations of skills. Attention should be given to the informative instructions on the front page of the examination paper. Candidates who do not show full method because they have used their calculator to perform key method steps, such as finding the value of the integral of a function for a particular set of limits without evidence of integration having been performed, will not gain full credit. Candidates need to take care to ensure that their calculator is in the appropriate mode when working with angles. Candidates should read each question carefully to ensure that their answers are given to at least the accuracy required. This is especially the case when finding angles.

General comments

The recall and application of techniques in order to solve problems, when necessary, was generally good. Some questions required the application of combinations of skills to answer them correctly. Many candidates were able to do this successfully. Some candidates would have benefitted from rereading the question, or reading the question more carefully, as the question they answered was not the question asked. This was evident in **Question 1** and **Question 10(a)** of this examination.

Many candidates offered sufficient and accurate method. When an exact value is required, the method used should produce an exact answer. Candidates should understand that, if they choose to use values that are rounded decimals in their working, their answer cannot be exact. This was evident in **Question 7(c)** and **Question 8(a)** in this examination. On occasion, candidates rounded working values too harshly. This resulted in a premature approximation error and a loss of final accuracy. In order for final answers to be accurate to three significant figures, for example, working values must be given to a greater accuracy. In this examination, this was evident in **Question 8(b)** and **Question 11**.

Candidates usually presented their work in a clear and logical manner. Some candidates used additional paper. This was useful as their work remained well presented and could be marked without difficulty. Candidates who did this usually added a comment in the answer space in their main script to indicate that their answer was written, or continued, elsewhere. This was very helpful and should be encouraged.

Most candidates attempted to answer the majority of questions. Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

A good number of candidates found this to be an accessible start to the examination paper. A very good number of fully correct solutions were seen. A few candidates would likely have benefitted from reading the question more carefully, as a common error was to find and use the gradient of *AB* or the gradient of a line perpendicular to *AB*. A few candidates made sign errors when finding the mid-point or when forming the equation.

Question 2

Many excellent and fully correct solutions were seen to this question. Most candidates were able to apply the correct method steps. A few candidates lost the final accuracy mark as their final steps were $42x = 7$, $x = 6$. A small number of candidates made incorrect initial steps such as $\log_5 8x + \log_5 7 - \log_5 2x = 2$ or $\dfrac{\log_5(8x+7)}{\log_5 2x} = 2$ or similar. No recovery was allowed following these errors.

Question 3

(a) The majority of candidates were able to answer this part of the question correctly.

(b) A good number of candidates were successful in this part. Those who were not, commonly found 144 or an incorrect multiple of 144, not taking into account all the possible cases.

(c) Candidates found this part to be the most challenging. A good number offered fully correct solutions, with those choosing to find the number of ways Cam and Dea were next to each other and subtracting this from 7! being the most successful. Candidates who chose to consider all the possible ways that they were not next to each other directly were sometimes successful, and some very clear and logical arguments were made. However this approach was more complex and candidates often omitted one or more possibilities.

Question 4

A good number of candidates demonstrated sound algebraic manipulation skills in this simultaneous equations question that included the solving of a disguised quadratic equation. Some excellent solutions were seen. A common error was to forget the negative square roots in the final step, or to find all four roots and then reject the negative ones as not being possible. A few other candidates gave their answers to 3 significant figures rather than the 3 decimal places required in the question. Weaker solutions typically involved issues with dividing $\left(\dfrac{3}{2x}\right)^2$ by 9. Commonly this became $\dfrac{81}{4x^2}$ in these cases. Similarly, some candidates rearranged the equation $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ as $y = \sqrt{9 - \dfrac{9x^2}{4}}$ and then incorrectly took the square root term by term. A few candidates found the correct values for x^2, but then labelled these as x and did not complete the solution.

Question 5

(a) A reasonable number of candidates were able to state both expressions for f(x), and did so efficiently. A good number of candidates were able to find one correct expression for f(x). Many of these attempted to rewrite this expression in a different form, misinterpreting what was needed. For example, it was not uncommon for candidates to attempt to complete the square.

(b) Many candidates found this question to be challenging. A reasonable number of candidates found the correct factors from the given roots, as expected. The key to success, in most cases, was to find the value of n using the constant in the polynomial and the information given. There were various ways that this could be done. Several candidates multiplied out their factorised form, with factors in terms of n, but made an error in doing so. Not many candidates used what was probably the most obvious method which was to equate –2 with the product $-1 \times (-n) \times \left(-(n+1)\right)$, to form a quadratic equation and solve for n. This resulted in a far easier expression to expand as the terms were simpler and the signs easier to manage. There were a small number of very neat solutions using the product of the roots and the sums of roots. Quite a few candidates formed 3 equations using the roots but made no progress beyond that.

Question 6

(a) (i) This part of the question was usually well-answered with most candidates able to write all four terms correctly. A few candidates made bracketing errors, relating to $3x$, that resulted in the final two terms being incorrect.

(ii) A good number of fully correct solutions were seen. These candidates deduced correctly that they needed to use $x = 0.01$ in their expression. Most showed that the result was a value that rounded to 1.23, although a few omitted this important step. Some candidates were unsure of how to use their answer to **part (a)(i)** and simply evaluated 1.03^7 on their calculator and rounded it.

(b) Candidates were slightly more successful in this part of the question. Many were successful in identifying the correct term by considering pairs of values where '4 $\times$ one value' and 'the other value' were the same. A few candidates wrote down the correct term but made errors in simplifying to find the final value. Some candidates simply wrote down some, or all, of the expansion without selecting a term. Some candidates used long, algebraic methods to find the powers for the correct term. Sometimes this resulted in an error. Whilst algebraic methods are usually preferable, as the values are integers and reasonably small in size, using trials or inspection is possible and quite sensible. These approaches may have reduced the errors made in some cases.

Question 7

(a) A very good number of candidates made a good start to this question, making the expected substitution and forming and factorising the resulting quadratic in $\tan\theta$. Quite a few candidates gave three solutions only, with –2.03 commonly omitted. Other candidates stated only the two solutions 1.11 and $-\frac{\pi}{4}$. Those candidates who did not use $1 + \tan^2\theta$ usually tried to use combinations of $\sin\theta$ and $\cos\theta$ with very limited success. Sometimes candidates resorted to doing this even after a correct first step. A few candidates had issues with rounding 1.107 to an acceptable accuracy, as 1.1 was fairly often stated for this angle without any greater accuracy being seen. A few candidates formed the correct quadratic in $\tan\theta$ but then just wrote down $\tan\theta = 2$ with no method shown and no other solution. This was not accepted as a method of solution. A small number of candidates worked in degrees and never gave values in radians. A few candidates converted their values to radians but made rounding errors in so doing. These errors could have been avoided by working in radians initially.

(b) Many good and clear solutions were seen to this part of the question. A few candidates made a correct pair of substitutions but were unable to cope with the final manipulation. This was far more common in those who were trying to simplify $\dfrac{\frac{\sin\phi}{\cos\phi}}{\sin\phi}$ which often became $\dfrac{\sin^2\phi}{\cos\phi}$. Candidates who wrote $\dfrac{1}{\sin\phi}\times\dfrac{\sin\phi}{\cos\phi}$ were far more successful. Some poor use of notation such as $\dfrac{\sin}{\cos}\phi$ was evident. This was not condoned for the accuracy mark. Some candidates treated the identity as an equation and multiplied across, which was not accepted. It is expected that candidates will work to show that the left-hand expression is equal to the right. A few candidates did not show their full method. For example, on occasion, the substitution of $\dfrac{\sin\phi}{\cos\phi}$ for $\tan\phi$ was not shown.

(c) Candidates found this to be the most challenging part of the question. A small number of candidates gave concise, neat and accurate solutions. A good number of candidates used a correct method but then omitted to take note of the appropriate sign and gave the answer $\frac{15}{8}$.

Various methods were possible, but as an exact answer was required, the method used needed to lead to an exact answer. Candidates who found a decimal value for x and then used a rounded decimal value to find $\cot x$ greatly reduced the level of difficulty of the question.

Question 8

(a) This question was quite challenging, although a good number of candidates still coped well with the problem solving and the combinations of skills needed to be successful. A few candidates would have benefitted from reading the question more carefully, as an exact form was required and some candidates resorted to using decimals at very early stages. A few candidates were careless with forming or solving the equation using the areas they had found. This usually resulted in the doubling or halving of the incorrect quantity. Weaker solutions typically assumed that a was half the radius or that triangle AOC was right-angled.

(b) Again, this was quite challenging. Most candidates managed to find the arc length correctly. Some candidates were confused by the values they needed to use in the cosine rule, with some candidates using the value of a instead of OC. Other candidates did not quote the cosine rule correctly. It is expected that the structure of the cosine rule is quoted correctly as this formula is given to candidates on page 2 of the examination paper. Those candidates who assumed that AOC was a right-angled triangle in **part (a)** tended to make the same incorrect assumption in this part. A few candidates made premature approximation errors by using working values that had been too harshly rounded. Some candidates did not sum three lengths to find their perimeter. Candidates should be aware that all key method steps should be shown. A few candidates could potentially have earned more marks because they listed values, which were incorrect, but did not show their sum.

Question 9

(a) The key to success in this question was to draw a reasonable velocity–time graph to represent the vehicle's motion. Candidates who drew a diagram were far more likely to be successful than those who did not. Although it was possible to earn full credit without a diagram, it was advisable and should be encouraged as good practice. A reasonable number of fully correct solutions were seen. A few candidates formed a partially correct equation. For example, a common error was to only include the triangular section of the trapezium, 70, and not the rectangular section, $10(w - 14)$.

Some candidates offered solutions based upon $\frac{458}{50}$.

(b) (i) This part of the question was very well answered with few incorrect answers seen.

(ii) This was also generally well answered. Most candidates were able to identify the critical values needed from the factorised form given and many of these were able to compose the correct inequality in t.

(iii) Some very accurate and excellent solutions were offered. In this part, candidates needed to interpret the information they had found in the previous part and deduce that the particle changed direction when $t = 4$. It was essential to this part that they found the displacement at the point where $t = 4$ and then calculated the displacement between 4 and 5 separately, changing the sign and then adding, or the equivalent to this in steps. Most candidates, however, did not understand the relevance of the information they had just found and simply integrated the expression for v between the limits of 0 and 5. A few candidates attempted to use $t = 4.5$, presumably from **part (a)**, in some way.

Question 10

(a) A very good proportion of candidates were able to earn the first 3 marks, finding the correct vector $\overrightarrow{OR}$. A good number of these continued to find the unit vector in this direction, but quite a few did not. Perhaps these candidates would have done better if they had reread the question. A few candidates made errors when finding $\overrightarrow{PQ}$, commonly summing the position vectors of the points P and Q instead of finding the correct difference. Other candidates incorrectly used $5\overrightarrow{PQ} = \overrightarrow{PR}$ or made sign or arithmetic errors in their solution.

(b) A reasonable number of candidates were able to form a correct vector $\overrightarrow{RS}$ in terms of λ. Some of these candidates constructed a correct proportion or used a second scalar k and correctly completed the solution. There were various successful approaches. Those who equated unit vectors and solved, needed to reject an extraneous solution and this was usually not done. Those who considered gradients were commonly successful, although some candidates needed to take more care as the gradient found was inverted. The most common error was to equate $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ and solve to find $\lambda = 16$.

Question 11

Some excellent solutions were seen to this question, with many candidates able to apply the combinations of problem-solving skills needed to produce an accurate and complete answer. Some candidates made premature approximation errors, often when finding the area of the rectangle, and lost the final accuracy mark. A few candidates omitted to show the integrated expression and this was not condoned for full credit. A small number of candidates differentiated the exponential term instead of integrating it and other candidates used incorrect limits. Weaker solutions made no attempt to do anything other than find the area between the curve and the x–axis for $x = 0$ to $x = 2$. Integration with respect to y, to find the area between the curve and the y-axis, is not expected for this course. Some candidates were attempting to integrate the given function of x between the limits of $y = 6.006...$ and $y = 26.08....$. This was far more likely to be a misunderstanding about the process of finding the plane area in relation to the x-axis, and what the limits represent, than an attempt to integrate with respect to y.

Question 12

The final question on the paper was challenging and some candidates made no attempt to answer.

(a) In this first step to solving a practical optimisation problem using calculus, candidates needed to apply skills they had already learned in order to derive an expression for the volume of the triangular prism in terms of h. A small number of candidates were able to derive a fully correct expression. Some candidates were able to find a correct expression for PQ in terms of h, and form a correct volume, but were unable to manipulate the expression to the required form. Others found a correct expression for PQ in terms of h, but were then unable to correctly recall how to find the volume of a prism. Other candidates found the area of triangle ABC and correctly used the area factor to form the expression for triangle PQR. The result for the volume quickly and neatly followed this. Those who were unable to form a correct scale factor often tried to use the proportional relationship $\frac{8}{12} = \frac{PQ}{h}$ or similar.

(b) A small number of fully correct solutions were seen in this part. Those candidates who had found a correct expression in **part (a)** generally continued to complete the problem successfully in this part. Candidates who used an expression from **part (a)** were also rewarded for correct derivatives and method, where possible.